# Quants



- 1. Nominal risk-free rate = Real risk-free rate + Expected inflation rate.
- 2. Additive Model: Nominal Rate = Inflation Premium + Real Rate
- 3. Multiplicative Model: (1 + Nominal Rate) = (1 + Inflation Rate) (1 + Real Rate)
- Required interest rate = Nominal risk-free rate+ default risk premium + liquidity premium +maturity risk premium.
- 5. Effective Annual Rate (EAR) = (1+ periodic rate)<sup>m</sup> -1

periodic rate = stated annual rate/m

m = no. of compounding periods per year

6. For continuous compounding, EAR =  $e^{rt} - 1$ 

7. Single Cash Flow: 
$$PV = \frac{FV}{(1+r)^n}$$
 or  $FV = PV (1 + r)^n$ 

8. FV = PV 
$$(1 + I/V)^N$$

9. 
$$PV_{perpetuity} = \frac{PMT}{I/y}$$

#### **Discounted Cash Flow Applications**

- 1. NPV = PV(inflows) PV(outflows)
- 2. Holding Period Return(HPR) =  $\frac{\text{Ending value-Beginning value}}{\text{Beginning value}} \text{ Or } \frac{P_1 P_0 + D}{P_0}$
- 3. Money Weighted Rate of Return (MWROR) = IRR (depends on magnitude and timing)
- 4. Time Weighted Rate of Return (TWROR)
  - =  $[(1 + r_1)(1 + r_2)....(1 + r_n)]^{1/n} 1$

Where,  $(1 + r_1) = HPR$ 

- 5. Bank Discount Yield (BDY)= $\frac{F-P}{F} \times \frac{360}{n}$
- 6. Holding Period Yield (HPY) =  $\frac{F-P}{P} \times 100$  Or  $\frac{P_1 P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} 1$
- 7. Effective Annual Yield (EAY)=  $(1 + HPY)^{\frac{365}{n}}$ -1
- 8. (Annualized HPY & annual compounding)

: HPY =  $(EAY + 1)^{n/365} - 1$ 

- 9. Money Market Yield (MMY)= HPY X  $\frac{360}{n}$  [Annual HPY in multiplicative fashion]
- 10. Bond Equivalent Yield (BEY) = 2 x semiannual discount rate (per annum compounded semiannually) =  $\left[(1 + EAY)^{\frac{1}{2}} 1\right] \times 2$

## Organizing, Visualizing, and Describing Data Statistical Measures of Asset Returns

- 1. Population mean ( $\mu$ ) =  $\frac{\sum_{i=1}^{N} X_i}{N}$ ; where N is population size Sample mean ( $\overline{X}$ ) =  $\frac{\sum_{i=1}^{N} X_i}{n}$ ; when n is sample size
- 2. Sum of mean deviations =  $\sum_{i=1}^{N} (x_i \overline{x}) = 0$

3. Geometric mean (GM) = 
$$\sqrt[n]{(x_1 * x_2 \dots x_n)}$$

Geometric mean return (R<sub>g</sub>): 1 + R<sub>g</sub> =  $\sqrt[n]{(1 + R_1) (1 + R_2) .... (1 + R_n)}$ 

 $AM \geq GM$  [AM - GM increase as the dispersion of the observations increase.]

AM = GM [When all observations are equal]

4. Harmonic mean (HM) =  $\frac{N}{\sum_{i=1}^{N} \frac{1}{x_i}}$  (average cost of shares purchase over time)

AM > GM > HM (dollar cost averaging uses investing same amount every time period in a share; average price will be lowest as HM is < AM or GM)

- 5. Ly = (n+1)  $\frac{y}{100}$  [Quartiles, Deciles and Percentiles]
- 6. Range = Maximum Value Minimum Value

7. Mean Absolute Deviation (MAD) = 
$$\frac{\sum_{i=1}^{N} |xi-\bar{x}|}{N} = \frac{\sum |x-\bar{x}|}{N}$$

8. Population variance,  $\sigma^2 = \frac{\sum_{i=1}^{N} (x-\mu)^2}{N}$ 

9. Population Standard Deviation (
$$\sigma$$
) =  $\sqrt{\frac{\sum_{i=1}^{N} (x-\mu)^{2}}{N}}$ 

 $\sigma > MAD$ 

10. S<sup>2</sup> = 
$$\frac{\sum_{i=1}^{N} (x-\bar{x})^2}{N-1}$$

11. K = 
$$\frac{\sum_{i=1}^{N} \mathbf{x} - \mu}{\sigma}$$

Standardizing a variable converts the mean into 0 and Standard Deviation into 1

12. Chebyshev's inequality / Bienaymé Chebyshev's Theorem

% of observations that lie within K standard deviation of mean is at least= $1 - \frac{1}{K^2}$ i.e., min Probability that variable will lie between $\mu \pm K\sigma = 1 - \frac{1}{K^2}$ (Applicable for all distribution) (K > 1)

- 13. Coefficient of variation (CV) =  $\frac{\sigma}{\mu}$ x 100 OR ( $\frac{S_x}{\bar{x}}$ x 100)
- 14. Sharpe ratio (Reward to variability ratio/SR) =  $\frac{\overline{R}_p R_f}{\sigma_p}$
- 15. Symmetrical: Mean = Median = Mode

Positive skewness: Mean > Median > Mode



# Probability Concepts Probability Trees and Conditional Expectations Portfolio Mathematics

- 1. **Probability** =  $\frac{\text{no of favourable outcomes}}{\text{total possible outcomes}}$
- 2.  $P(A) \Rightarrow$  Marginal / Unconditional Probability
  - $P(A \cap B) \Rightarrow$  Joint Probability A and B
  - $P(A \cup B) \Rightarrow$  Total Probability A or B
  - $P(B \mid A) \Rightarrow$  Conditional Probability of B given that A has occurred
- 3.  $P(A | B) = \frac{P(A \cap B)}{P(B)}$

Or  $P(A \cap B) = P(A \mid B)$ . P(B)

(Multiplication rule of probability)

4.  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 

(Addition rule)

5. For, mutually exclusive event,  $P(A \cap B) = 0$ 

For, independent event,  $P(A \cap B) = P(A) P(B)$ 

Also, P(A | B) = P(A) or P(B | A) = P(B)

6.  $P(R) = P(R | S_1) \times P(S_1) + P(R | S_2) \times P(S_2) + P(R | S_n) \times P(S_n)$ 

Where  $\{S_1\,,S_2\,\ldots,S_n\}$  is mutually exclusive & exhaustive [total probability rule]

7. BAYES' THEOREM - Posterior Probability

# Common Probability Distribution Appendices

#### **Simulation Methods**

- 1. Expected value E(x) = Weighted average of all possible outcomes  $\sum PX$
- 2.  $\sigma^2 = \sum P. (X \overline{X})^2$
- 3. Cov  $(R_A, R_B) = \sum P(S) \times [R_A E(R_A)][R_B E(R_B)]$
- 4. Correlation  $(R_i, R_j) = \frac{Cov(R_i, R_j)}{\sigma_{R_i^*}\sigma_{R_i}}$
- 5. Weight ( $W_i$ ) =  $\frac{MV \text{ of investment in Asseet}}{MV \text{ of the portfolio}}$
- 6. Expected value of portfolio composed of n asset :  $E(R_P) = W_1E(R_1) + W_2E(R_2) + \dots + W_nE(R_n)$
- 7. Var  $(R_p)$  for a two-asset portfolio =  $W_A \sigma_{R_B}^2 + W_B \sigma_{R_B}^2 + 2W_A W_B \operatorname{cov} (R_A R_B)$ Variance of n asset portfolio will have n(n-1)/2

Unique cov  $(R_A, R_B)$  as cov  $(R_A, R_B) = cov (R_B, R_A)$ 

- 8. Var (R<sub>P</sub>) for a 3 asset portfolio =  $W_A^2 \sigma_{R_A}^2 + W_B^2 \sigma_{R_B}^2 + W_C^2 \sigma_{R_C}^2 + 2 [W_A W_B \operatorname{cov}(R_A R_B) + W_A W_C \operatorname{cov}(R_A R_C) + W_B W_C \operatorname{cov}(R_B R_C)]$ Cov (R<sub>A</sub>R<sub>A</sub>) = Variance R<sub>A</sub> or  $\sigma_{R_A}^2$
- 9. Probability of function P(x) = P (X=x) (for discrete variables)

$$\Rightarrow$$
 0 $\leq$  p (x)  $\leq$  1

$$\Rightarrow$$
  $\Sigma$  P (x) = 1

- 10. Cumulative distribution function CDF F (x) = P (X  $\leq$  x)
- 11. Bernoulli trials:  $P(x) = n_{C_x} p^x (1-p)^{n-x}$
- 12. In Binominal Distribution,

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Df, P < 0.5 + ve Skewness
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- P = 0.5 Symmetrical
- P >0.5 -ve Skewness

Expected value of a Binominal Random Variable  $\Rightarrow E(X) = np$ 

Variance of a Binominal Random Variable  $\Rightarrow$  Variance of X = np(1-p)

Effective annual rate  $\Rightarrow e^{R_{cc}}$ 

 $\ln \left(\frac{S_1}{S_0}\right)$  = ln (1+HPR) = R<sub>cc</sub> (rate of continuous compounding)

13.  $Z = \frac{\text{observation-population mean}}{\text{S.D.}} = \frac{\bar{x} - \mu}{\sigma}$ 

90% confidence internal,  $\bar{x}$  - 1.65s to  $\bar{x}$  + 1.65s

95% confidence internal,  $\overline{x}$  - 1.96s to  $\overline{x}$  + 1.96s

99% confidence internal,  $\overline{x}$  - 2.58s to  $\ \overline{x}$  + 2.58s

14. Roy's Safety-First Ratio (SFR) =  $\frac{E(R_P)-R_{min}}{\sigma_P}$  (higher the better)

R<sub>min</sub>= threshold level

If threshold level = Risk free rate of return, i.e.  $R_{min} = R_f$ , SFR = Sharpe's Ratio

# Sampling and Estimation Estimation and Inference

- 1. Sample error of the mean = Sample mean Population mean
  - $= \overline{x} \mu$
- 2. Standard error of sample mean ( $\sigma_{\bar{x}})$ 
  - =  $\frac{\sigma}{\sqrt{n}}$  (If  $\sigma$  is known)
  - =  $\frac{s}{\sqrt{n}}$  (If  $\sigma$  is not known)
- 3. Confidence Interval:  $\bar{x} \pm Z\alpha_{/2} \frac{\sigma}{\sqrt{n}}$

 $\alpha$ - Level of significance (for 3 distribution)

4. Confidence Interval:  $\bar{x} \pm t \alpha_{/2} \frac{s}{\sqrt{n}} [\sigma \text{ not known}]$ 

t is calculated as df(n-1)  $\rightarrow \frac{\alpha}{2}$ 

# Basics of Hypothesis Testing Hypothesis Testing Parametric and Non-Parametric Tests of Independence

1. Equality of mean (independent samples)

t statistic of 
$$\overline{x}_1-\overline{x}_2$$
 =  $\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sigma_{X_1}-X_2}$ 

Where, 
$$\sigma_{x_1-x_2} = \sqrt{\frac{S_{P^2}}{n_1} + \frac{S_{P^2}}{n_2}}$$
  
$$S_{p^2}^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

2. Equality of mean: Dependent Samples

$$\dagger = \frac{\overline{d} - \mu}{s_{\overline{d}}}$$

Where,  $\overline{d}$  = Mean of differences between the samples;  $S_{\overline{d}}=\frac{Standard\ deviation\ of\ the\ differences}{\sqrt{n}}$ 

3. Testing of variance (Chi square statistic):

X<sup>2</sup> Statistic = 
$$\frac{(n-1)S^2}{\sigma^2}$$

Where, S<sup>2</sup> = Sample variance

 $\sigma^2$  = Hypothesized value for sample variance.

4. Testing of equality of variance (F distribution) F statistic =  $\frac{S_1^2}{S_2^2}$ 

# **Economics**



#### The Firm & Market Structures

1. Perfect Competition  $\rightarrow$  Firm faces infinitely elastic demand

MR = AR = P = D

(Price is determined by the market supply and demand.)

2. MR = 
$$P(1 - \frac{1}{E_n})$$

- 3. HHI =  $\Sigma$ (market share)<sup>2</sup>
- 4.  $N_{firm} = \sum (market share)$



- 1. GDP Deflator =  $\frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100$
- 2. Per capita Real GDP =  $\frac{\text{Real GDP}}{\text{Population}}$
- 3. GDP:

Under Expenditure Approach

C+I+G+(X-M)

Under Income Approach

NI + Depn (CCA) + Statistical Discrepancy or C+S+T

#### 4. National Income

Compensation of Employees (Wages/COE) + Interest Income + Rent + Corporate & Govt.

Enterprise Profit before Taxes+ Unincorporated Business Net Income + Indirect Business

Taxes - Subsidies

#### 5. Personal Income

National Income + Transfer payment to household

- Taxes (Indirect Business & Corporate)
- Undistributed corporate profits
- 6. Potential GDP = aggregate working hours x labor paid

Growth in potential GDP = growth in labor force + growth in labor productivity

 $\begin{array}{cccc} S &= I \\ (Savings) & (Investment) \end{array} + \begin{array}{ccc} (G-T) & (X-M) \\ + & (Fiscal + & (Trade \\ balance) & balance) \end{array}$ 

- 7. Personal Disposable Income = Personal Income Personal Taxes.
- 8. Sustainability of Economic Growth:

Potential GDP = aggregate hours worked x labor productivity

Growth in Potential GDP = growth in labor force + growth in labor productivity.

9. Production Function:

Total  $Y = A \times f(L, K)$ 

10. Production per worker basis:

$$Y/L = A \times f\left(\frac{k}{L}\right)$$



- 1. High powered money = Fed Currency + Reserve + Govt. money (coin)
- 2. M = money supply = mH
- 3. Money created =  $\frac{\text{new deposit}}{\text{reserve requirement}}$
- 4. Money multiplier =  $\frac{1}{\text{reserve ratio}}$  = m
- 5. money supply(M) x velocity(V) = price(P) x real output(Y) [MV = PY]
- 6. The Fisher effect:

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R_{Nom} = R_{Real} + E[I]
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For risky securities:

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R_{Nom} = R_{Real} + E[I] + RP
```

- 7. Nominal = real + inflation
  - (1+nominal) = (1+ real) (1+ inflation) + risk premium
- 8. Neutral int. rate = real tread rate of growth + infl. Target
- 9. Policy rate = neutral +  $\frac{1}{2}$  (actual target) growth +  $\frac{1}{2}$  (actual target) inflation
- 10. Fiscal multiplier =  $\frac{1}{1-MPC(1-t)}$



## Currency Exchange Rates Capital Flows and the FX Market Exchange Rate Calculations

1. Real exchange rate (d/f) = nominal exchange rate  $x \left[ \frac{CPI_{foreign}}{CPI_{domestic}} \right]$ 

2. 
$$R_{P/B} = S_{P/B} (P_B/P_A)$$

- 3. New Exchange Rate = old exchange rate  $\left(\frac{1+\ln fA}{1+\ln fB}\right)$
- 4. Cross Rate =  $\frac{MXN}{AUD} = \frac{MXN}{USD} \times \frac{USD}{AUD}$

5. Interest Rate Parity (IRP) = 
$$S_{A/B} \times \left(\frac{1+iA}{1+iB}\right)^{T}$$

6. Marshall - Lerner condition:

 $W_X \varepsilon_X + W_M (\varepsilon_M - 1) > 0$ 

7. The Absorption Approach:

BT (Balance of Trade) = Y (Income) - E (Expense)

# FRA



1. Balance Sheet - Financial position - at a point in time

Assets = liabilities + owners' equity.

## Income Statement Analyzing Income Statements

- 1. Revenues Expenses = Net Income
- 2. Net Income = Revenues Ordinary Expenses + Other Income Other Expenses + Gains Losses
- 3. Profit = Cash receive during period x  $\frac{\text{Total Expected Profit}}{\text{Sales}}$
- 4. Straight line Depreciation:

Cost–residual value Useful Life

5. Double Declining Depreciation:

 $\frac{2}{\text{Usefullife}}$  (Cost - accumulated Depreciation)[\* salvage value not to be considered here]

- 6. Basic EPS =  $\frac{\text{EAFESH}}{\text{wtd. Average of no.of shares}}$
- 7. Diluted EPS =  $\frac{[PAT-pref.div]+conveitble prefeved div+convertible in(1-t)}{Wtd.average no.of shares + shares from conversion of convertible preference share debt + Shares from conversion of convertible preference shares from of options | wairants$
- 8. Comprehensive Income = Net Income (PAT) + Other Comprehensive Income [OCI]
- 9. Gross profit margin =  $\frac{GP}{Revence/sales}$
- 10. Net profit margin =  $\frac{NP}{sales}$



# Cash Flow Statements Analyzing Statements of Cash Flows I Analyzing Statements of Cash Flows II

- 1. FCFF = NI + Interest [1-tax] + Depreciation Working Capital Investment FC Investment
- 2. FCFE = CFO FC Inv + Net Borrowing

#### Performance Ratio:





```
Solvency:

26. Debt-to-Equity = \frac{\text{Total Debt}}{\text{Total shareholders equity}}

27. Debt-to-Capital = \frac{\text{total debt}}{\text{total debt+total equity}}

28. Debt-to-Assets = \frac{\text{total Debt}}{\text{Total Assets}}

29. Financial leverage = \frac{\text{Average Assets}}{\text{Average Equity}} = A/E

30. Interest coverage = \frac{\text{Earning Before Interest & Taxes}}{\text{interest payments}}

31. Fixed charge coverage = \frac{\text{EBIT+lease payments}}{\text{interest +lease payments}}

DuPont System of Analysis:

1. ROE = \frac{\text{Net Income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Asset}} \times \frac{\text{Assets}}{\text{Equity}}

2. ROE = \frac{\text{PAT}}{\text{PBT}} \times \frac{\text{PBT}}{\text{EBIT}} \times \frac{\text{EBIT}}{\text{SALES}} \times \frac{\text{SALES}}{\text{ASSETS}} \times \frac{\text{A}}{\text{E}}

Dividends:

3. G = \text{RR} \times \text{ROE}

4. Retention Rate = 1 - Dividend Payout Ratio
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5. Dividend Payout Ratio = Dividends Net income available to common shareholder

### Inventories Analysis of Inventories

- 1. Cost of Goods Sold (COGS) = Beginning inventory + purchases ending inventory
- 2. FIFO COGS = LIFO COGS (ending LIFO reserve beginning LIFO reserve)
- 3. Weighted Average Cost -

Cost per unit is calculated using this formula =

Total Cost of Goods Available for sale (Opening Inventory + Purchases) Total Quality Available for sale

4. Ending Inventory = Beginning Inventory + Purchases - COGS



# Income Taxes Analysis of Income Taxes

- 1. Income tax Expense = taxes payable +  $\Delta DTL \Delta DTA$
- 2. DTA = Tax expense < Tax payable
- 3. DTL = Tax expense < Tax payable
- Interest Expense = (the market rate at issue) x (the balance sheet value of the liability at the beginning of the period)
- 5. Effective Tax Rate =  $\frac{\text{Income Tax Expense}}{\text{Pretax Income}}$



# Corporate Finance



# Equity Investments

Equity Valuation: Concepts and Basic Tools

1. Dividend discount model:

$$\mathbf{V}_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^2}$$

- 2. Preferred stock value =  $\frac{D_P}{(1+K_P)^1} + \frac{D_P}{(1+K_P)^2} + \dots + \frac{D_P}{(1+K_P)^{\alpha}} = \frac{D_P}{K_P}$
- 3. FCFE = Net Income + Depreciation Increase in working Capital Fixed Capital Investment debt principal repayments + new debt issues

4. 
$$P_0 = \frac{D_1 + P_1}{1 + K_e}$$

5. 
$$P_0 = \frac{D_1}{K_e - g} = \frac{D_0 (1+g)}{K_e - g} [K_e > g]$$

g = constant growth rate

Also,  $K_e = \frac{D_1}{P_0} + g$ 

K<sub>e</sub> = investor required rate

6. Gordon growth model:

$$V_{0} = \frac{D_{0}(1+g_{c})}{1+K_{e}} + \frac{D_{0}(1+g_{c})^{2}}{(1+K_{e})^{2}} + \frac{D_{0}(1+g_{c})^{3}}{(1+K_{e})^{3}} + \dots + \frac{D_{0}(1+g_{c})^{\infty}}{(1+K_{e})^{\infty}}$$

7. 
$$P_0 = \frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \frac{\frac{D_4}{K_e-g}}{(1+K_e)^3}; \frac{D_4}{K_e-g} = P_4$$

when the growth rate of dividend is constant:

$$V_0 = \frac{D_0(1+g_c)}{K_e - g_c} = \frac{D_1}{K_e - g_c}$$

8. Multistage dividend discount model:

Value = 
$$\frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \dots + \frac{D_n}{(1+K_e)^n} + \frac{P_n}{(1+K_e)^n}$$
  
Where ,  $P_n = \frac{D_{n+1}}{K_e - g}$ 

9.  $\frac{P_0}{E_1}$  = leading /expected PE Ratio

 $\frac{D_1/E_1}{K_e-g}$  [Dividend Payout Ratio =D $_1/E_1$ ]

 $\left[\frac{P_0}{E_0} - \text{lagging} / \text{historical PE ratio}\right]$ 

10. Sustainable growth = ROE x (1 - dividend payout ratio)

11. P/BV ratio (Price / book Value Ratio) =  $\frac{\text{market value of equity}}{\text{book value of equity}} = \frac{\text{market price per share}}{\text{book value per share}}$ 

Book value of equity = (total assets - total liabilities) - preferred stock

- 12. P/S Ratio (Price to Sale Ratio) =  $\frac{\text{Market Value of Equity}}{\text{Total Sales}}$
- 13. Enterprise Value = Market value of stocks + Market value of debt Cash and short term investments.

### Market Organization and Structure

1. Margin call price =  $P_0\left(\frac{1-\text{Initial margin}}{1-\text{maintenance margin}}\right)$   $P_0$  = Initial purchase price



### **Overview of Equity Securities**

1. ROE = 
$$\frac{NI_t}{(BV_1+BV_{t-1})/2}$$
  
ROE =  $\frac{NI_t}{BV_{t-1}/2}$ 

2. DDM: 
$$R_e = \frac{D_1}{P_o} + g$$

4. Market Price 
$$\frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots$$

# Fixed Income



- 1. Conversion Ratio =  $\frac{\text{Par Value of the Bond}}{\text{Conversion Price}}$
- 2. **Conversion Value =** Conversion Ratio × Current Market Price of a Common Share



- 10. Current Yield =  $\frac{\text{anual coupon payment}}{\text{Bond price}}$
- 11. Option Value = Zspread OAS (Option Adjusted Spread)

### Fixed-Income Securitization Asset-Backed Security (ABS) Instrument and Market Features Mortgage-Backed Security (MBS) Instrument and Market Features

- 1. Debt-to-service =  $\frac{NOI}{debt \text{ service}}$ ; where debt service = principal + interest
- 2. Loan-to-value =  $\frac{\text{current mortgage amount}}{\text{current appraised value}}$
- 3. Annualized Conditional prepayment rate (CPR) = 1 (1-SMM)<sup>12</sup>
- 4. Single month mortality (SMM) = 1 (1-CPR) <sup>1/12</sup>

Interest Rate Risk and Return Yield-Based Bond Duration Measures and Properties Yield-Based Bond Convexity and Portfolio Properties Curve-Based and Empirical Fixed-Income Risk Measures

- 1. Macaulay's Duration = wtd. Average of time, where W= PV of  $CF = \frac{\Sigma WX}{\Sigma w}$ Modified Duration =  $\frac{\text{macaulay's duration}}{1+\text{ytm/m}}$   $\longrightarrow$  Compounding frequency
- 2. Modification Duration  $\cap \cong E.D.$  [only for option free bonds]
- 3. Effective Duration (E.D.) =  $\frac{P_2 P_1}{2\Delta Y P_0}$ ;  $\Delta Y$ = change in YTM
- 4. Effective Convexity (E.C.) =  $\frac{P_2 + P_1 2P_0}{P_0 (\Delta Y)^2}$
- 5. Convexity adjustment =  $\frac{1}{2} \times EC \times (\Delta Y)^2$
- 6. Money duration = annual modification duration x full price of bond position
- 7. Portfolio duration =  $\sum w_i d_i$ ;  $W_i$ = market value rates
- 8. Change in full bond price = (-E.D.  $\times \Delta Y$ ) +  $\frac{1}{2} \times Ec \times (\Delta Y)^2$
- 9. Duration gap = Macaulay's duration investment horizon
- 10. Price Value of a Basis Point (PVBP) =  $\frac{P_2 P_1}{2}$

### Credit Risk

### Credit Analysis for Government Issuers Credit Analysis for Corporate Issuers

- 1. Expected loss = exposure × prob of default (default risk) × loss severity (1-RR)
- 2. Credit risk = default risk + loss severity (1-RR)
- 3. Yield Spread =  $YTM_{Credit risky bond} YTM_{risk free bond}$  Or,

Yield spread = liquidity premium + credit spread (will widen) affected by 2 factors:

- credit wordiness ↓(credit migration / downgrade risk)
- Market liquidity risk.
- 4. Enterprise Value = Equity + Debt Cash and Marketable Securities
- 5. Leverage Ratios:
  - Debt/Capital
  - Debt/EBITDA
  - FFO/Debt
  - FCF after dividends/Debt
- 6. Coverage Ratios:
  - EBITDA/Interest Expense
  - EBIT/Interest Expense

# Derivatives



### Forward Commitment and Contingent Claim Features and Instruments

1. Forward Price = Spot + Interest Cost + Storage Cost -Benefits.

Spot
$$(1 + R_F)^T$$
 + FV(Storage – Benefit)

2. The forward price of an asset to be delivered at time T is:

 $F_0(T) = S_0(1 + Rf)^T$ 

3. The value of a forward contract is zero at initiation:  $V_t(T) = S_t - F_0(T) / (1+R_f)^{T-t}$ 

4. Payoff to FRA =  $\frac{(\text{market rate-contracted rate}) \times NPx\frac{n}{12}}{1 + (\text{market rate } x\frac{n}{12})} \times \frac{n}{360}$  if no. of days

5. U = up factor

$$D = \frac{1}{11}$$

Probability risk neutral =  $\lambda u = \frac{(1+Rf)^T - D}{U-D}$ 

Call option value = ( $\lambda u \times C_1^+$ ) + ( $\lambda D \times C_1^-$ )

:. Today value = (co) =  $\frac{\text{call option value}}{(1+Rf)}$ 

- 6. Option premium = intrinsic value + time value
- 7. Put call parity =  $C_o + \frac{X}{(1+RF)^T} = P_o + S_o$
- 8. Put call forward parity =  $C_o + \frac{X}{(1+RF)^T} = P_o + \frac{F}{(1+RF)^T}$  $\rightarrow C_o - P_o = \frac{F-X}{(1+RF)^T}$

# Introduction to Alternative Investments



# Portfolio

### Portfolio management: An Overview

1. Diversification Ratio =  $\frac{\text{Risk of equally weighted portfolio of 'n' securities}}{\text{Risk of single security at random from 'n' securities}}$ 



- 1. Holding period return =  $\frac{\text{end of period value}}{\text{beginning of period value}} 1$
- 2. Arithmetic mean return =  $\frac{(R_1+R_2+R_3+\dots+R_n)}{n}$
- 3. Geometric mean return =  $\sqrt[n]{(1+R_1) \times (1+R_2) \times (1+R_3) ... \times (1+R_n)} 1$
- 4. Population variance:  $\sigma^2 = \frac{\sum (x-\bar{x})^2}{n}$
- 5. Sample variance:  $\sigma^2 = \frac{\sum (x \bar{x})^2}{n-1}$
- 6. Cov =  $\sum (X E_x) (Y E_y) \times P$
- 7.  $R_p = \sum W_i R_i$  and  $E_{R_p} = \sum W_i E_{R_i}$
- 8.  $\operatorname{Cov}_{1,2} = \frac{\sum (R_{t,1} \overline{R}_1) (R_{t,2} \overline{R}_2)}{n-1} = \frac{\sum (x \overline{x})(y \overline{y})}{n-1}$

9. 
$$\rho_{x,y} = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$$

10. 
$$\sigma_{\rm P} = \sqrt{\sigma_{\rm P}^2} = \sqrt{\sum w^2 \sigma^2 \sum w_i w_j \text{Cov}_{i,j}}$$

$$\sigma_{\rm P} = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 r_{1,2}}$$

$$\sigma_{\rm P} = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}_{1,2}}$$

- $\sigma_{P} = \sqrt{w_{1}^{2}\sigma_{1}^{2} + w_{2}^{2}\sigma_{2}^{2} + w_{3}^{2}\sigma_{3}^{2} + 2w_{1}w_{2}Cov_{1,2} + 2w_{2}w_{3}Cov_{2,3} + 2w_{1}w_{3}Cov_{1,3}}$
- 11. P/L from securities <u>- Commission and other Brokerage Expenses</u> Gross Return <u>- Management and Administration Fees</u>

Aversion (-ve)

Risk (-ve)

0

**Risk Neutral** 

Net Return

Utility

(-ve)

**Risk Seeking** 

- 12. After tax return = Pre-tax return (1-t)
- 13. Leveraged Return =  $\frac{\frac{\text{gain}}{\text{loss}}\text{on Total Investment}}{\text{Investor's Cash Investment}}$ 14. Investor's Utility Function:  $U = E(R) - \frac{1}{2}A\sigma^{2}$

**↓** 

(+ve)

Return Risk



(+ve)

**Risk Averse** 

# Portfolio Risk and Return: Part II

1. 
$$E(R_{P}) = (1 - w_{m})R_{f} + w_{m}R_{m}$$

$$= R_{f} + w_{m} (R_{m} - R_{f})$$

$$\sigma_{P} = \sqrt{(1 - w_{m})^{2}\sigma R_{f}^{2} + w_{m}^{2}\sigma_{m}^{2} + 2(1 - w_{m})\sigma_{Rf}\sigma_{Rm}r_{Rf}, R_{m}}$$
For  $R_{f}$ ,  $\sigma = 0$  and  $Cov = 0$ 

$$\therefore \sigma_{P} = \sqrt{w_{m}^{2}\sigma_{m}^{2}} = \sigma_{m}w_{m}$$

$$\therefore w_{m} = \frac{\sigma_{P}}{\sigma_{m}}$$
2. Capital market line:  
 $E(R_{P}) = R_{f} + \sigma_{P} \left[\frac{R_{m} - R_{f}}{\sigma_{m}^{2}}\right]$ 
3.  $E(R_{i}) = R_{f} + \frac{E(R_{m}) - R_{i}}{\sigma_{m}^{2}} \times Cov_{i,m}$ 
Or  $E(R_{i}) = R_{f} + \frac{Cov_{i,m}}{\sigma_{m}^{2}} [E(R_{m}) - R_{t}]$ 
4.  $\beta = \frac{Cov_{i,m}}{\sigma_{m}^{2}} = r \frac{\sigma_{i}}{\sigma_{m}}$ 
5. Market Model:  
 $R_{i} = \alpha_{i} + \beta_{i}R_{m} + \varepsilon_{i}$ 
6. Total risk = systematic risk + unsystematic risk
7. Single factor model:  
 $E(R_{i}) - R_{f} = \beta_{i} \times [E(R_{m}) - R_{f}]$ 
8. Risk free portfolio:  $W_{A} = \frac{\sigma_{B}}{\sigma_{A} + \sigma_{B}}$ 
9. Security Market Line:  
 $R_{e} = R_{f} + \frac{Cov_{i,mkt}}{\sigma_{m}^{2}}(R_{m} - R_{f})$ 
10.  $M - Squared = (R_{p} - R_{f})\frac{\sigma_{m}}{\sigma_{p}} - (R_{m} - R_{f})$ 
11. Sharpe Ratio  $= \frac{R_{p} - R_{f}}{\sigma_{p}}$ 

- 13. Jensen's Alpha =  $R_p CAPM$